Holographic elements with high efficiency and low aberrations for helmet displays

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A recursive design technique for forming holographic elements which can be incorporated into helmet displays has been developed. To ensure that the holographic elements have low aberrations and high diffraction efficiency they must be recorded with complex wavefronts. These wavefronts are derived from relatively simple intermediate holograms whose readout geometry and wavelength differ from those during recording. The overall recursive design technique is illustrated for a single holographic helmet display element having resolution of less than 0.8 mrad and diffraction efficiency of more than 80 percent over a field of view of $16^\circ \times 16^\circ$.

I. Introduction

Holographic optical elements (HOEs) have several advantages over conventional elements. They are lighter, more compact, and can be formed and replicated with relative ease. Even more important is the fact that they can sometimes perform complex optical operations which cannot be achieved with conventional optics. In general, the HOEs must be used with a monochromatic light sources.

An important application is the use of a HOE as an imaging lens and combiner for the helmet mounted display. Here a 2-D monochromatic display is imaged to infinity and reflected into a pilot's eye. The display can be imaged directly from a CRT or indirectly through a relay lens or an optical fiber bundle. The display is comprised of an array of points whose geometry at readout will differ from that at recording. Consequently, the imaged array will contain aberrations that decrease the image quality. In addition, the Bragg condition is not fulfilled for all points, so that the diffraction efficiency of the element is relatively low.

Recently there have been several proposals for designing imaging holographic lenses with improved performance. In these designs, aspherical rather than simple spherical waves, are used for recording the HOEs. The aspherical waves were derived from conventional optics, or from computer-generated holograms (CGH). Unfortunately, such approaches for obtaining the aspherical waves must rely on fairly complicated and costly components and equipment. Moreover, the aberrations are not completely corrected during recording, so that some corrective optics are necessary for readout; as a result the overall helmet display unit becomes more complex and cumbersome.

This paper presents a different approach, based on recursive techniques, for designing a holographic helmet display having improved performance. Here the aspheric waves for recording the final desired imaging element are derived from intermediate conventional holograms. In the following, we shall outline the recursive approach for designing a reflective holographic imaging element in a helmet display and then illustrate it by designing and testing a specific element having a resolution of less than 0.8 mrad and a diffraction efficiency of more than 80% over a field of view (FOV) of $16^\circ \times 16^\circ$.

II. Aberration Analysis

The diffraction from a hologram can be readily described by the phase of the participating recording and readout wavefronts. Specifically, the phase of the image wave, $\phi_i$, will, in general, be

$$\phi_i = \phi_d + (\phi_o - \phi_r),$$

where $i, c, o$, and $r$ are the indices of the image, reconstruction, object, and reference waves, respectively, and the $\pm$ sign refers to the diffracted positive first order $(+)$ or the diffracted negative first order $(-)$ of the hologram. When the desired Gaussian phase $\phi_d$ differs from the actual image phase $\phi_i$, we encounter some aberrations. For the geometries where the reference, object, and reconstruction waves are simple...
spherical waves, it is possible to describe the aberrations with a power series. Following the notation of Latta\(^9\) (according to the calculations for off-axis hologram third-order aberrations by Champagne\(^1\)), the Gaussian image properties of the hologram, because of wave matching in the meridional plane, are found as

\[
\sin \alpha_q = \sin \alpha_o \pm \mu (\sin \alpha_o - \sin \alpha_r), \tag{2}
\]
\[
\cos \alpha \sin \beta_1 = \cos \alpha_o \sin \beta_o \pm \mu (\cos \alpha_o \sin \beta_o - \cos \alpha \sin \beta_1), \tag{3}
\]
\[
\frac{1}{R_i} = \frac{1}{R_e} \pm \mu \left( \frac{1}{R_o} - \frac{1}{R_r} \right), \tag{4}
\]

where, as shown in Fig. 1, \(R_q\) (\(q = c, o, r\)) is the distance between the respective point source and the center of the hologram, \(\alpha_q\) is the angle between \(R\) and \(\xi - \zeta\) plane, \(\beta_q\) is the angle between the projection of \(R_q\) on the \(\xi - \zeta\) plane and the \(\xi\) axis, \(\mu\) is the ratio between the readout and the recording wavelengths (i.e. \(\lambda_o/\lambda_o\)), and the \(\pm\) notation, as before, denotes the diffracted positive and negative orders.

The wavefront deviation from the Gaussian sphere may be written as

\[
\Delta = \Delta_F + \Delta_S + \Delta_C + \Delta_A, \tag{5}
\]

where the field curvature is

\[
\Delta_F = \frac{2\pi}{\lambda_o} \frac{1}{2} F(\xi^2 + \zeta^2), \tag{6}
\]
the spherical aberration is

\[
\Delta_S = \frac{2\pi}{\lambda_o} \frac{1}{8} G(\xi^2 + \zeta^2)^2, \tag{7}
\]
the coma is

\[
\Delta_C = -\frac{2\pi}{\lambda_o} \frac{1}{2} \left[ Cx(\xi^2 + \zeta^2) + Cy(\eta^2 + \zeta^2) \right], \tag{8}
\]
and the astigmatism is

\[
\Delta_A = \frac{2\pi}{\lambda_o} \left( \frac{1}{2} Ax \xi^2 + \frac{1}{2} Ay \eta^2 + Axy \xi \eta \right). \tag{9}
\]

The terms \(F, S, Cx, Cy, Ax, Ay\) and \(Axy\) are defined by the equations

\[
F = \frac{1}{R_e} - \frac{1}{R_l} \pm \mu \left( \frac{1}{R_o} - \frac{1}{R_r} \right), \tag{10}
\]
\[
S = \frac{1}{R_i^2} - \frac{1}{R_l^2} \pm \mu \left( \frac{1}{R_o^2} - \frac{1}{R_r^2} \right), \tag{11}
\]
\[
Cy = \frac{\sin \alpha_o}{R_o^2} \frac{\sin \beta_o}{R_o^2} \pm \mu \left( \frac{\sin \alpha_o}{R_o^2} - \frac{\sin \alpha_r}{R_r^2} \right), \tag{12}
\]
\[
Cx = \frac{\cos \alpha_o \sin \beta_o}{R_o^2} \frac{\cos \alpha_r \sin \beta_r}{R_r^2} \pm \mu \left( \frac{\cos \alpha_o \sin \beta_o}{R_o^2} - \frac{\cos \alpha_r \sin \beta_r}{R_r^2} \right), \tag{13}
\]
\[
Cy = \frac{\sin^2 \alpha_o}{R_o^2} \frac{\sin^2 \alpha_r}{R_r^2} \pm \mu \left( \frac{\sin^2 \alpha_o}{R_o^2} - \frac{\sin^2 \alpha_r}{R_r^2} \right), \tag{14}
\]
\[
Ax = \frac{\cos^2 \alpha_o \sin^2 \beta_o}{R_o^2} \frac{\cos^2 \alpha_r \sin^2 \beta_r}{R_r^2} \pm \mu \left( \frac{\cos^2 \alpha_o \sin^2 \beta_o}{R_o^2} - \frac{\cos^2 \alpha_r \sin^2 \beta_r}{R_r^2} \right), \tag{15}
\]
\[
Axy = \frac{\cos \alpha_o \sin \alpha_r \sin \beta_o}{R_o} \frac{\cos \alpha_r \sin \alpha_o \sin \beta_r}{R_r} \pm \mu \left( \frac{\cos \alpha_o \sin \alpha_r \sin \beta_o}{R_o} - \frac{\cos \alpha_r \sin \alpha_o \sin \beta_r}{R_r} \right), \tag{16}
\]

where \(R_l\) is the distance between the hologram and the actual location of the image plane. When imaging only one point, it is best to choose that \(R_i = R_l\), but such a choice is not suitable when imaging more than one point.

The geometry for recording and readout of a simple reflective holographic element for a helmet display is shown in Fig. 2. For recording, Fig. 2(a), the object wave is an off-axis spherical wave at a distance \(R_o\) and an angle \(\alpha_o\) from the center of the hologram, whereas the reference wave is a plane wave at an angle \(\beta_o\) arriving from the opposite side to the hologram plane. For readout, Fig. 2(b), a display is inserted at an angle \(\theta\) which is the same as \(\alpha_o\), and at a distance \(R_{dis}\) from the center of the hologram. An observer, located at a distance \(R_{eye}\) and an angle \(\alpha_{eye}\) (which is opposite to \(\beta_o\)), sees a collimated image of the display. Even though the real reconstruction waves emerge from the display plane and are imaged by the hologram onto the eye, it is better (for the sake of simplifying the aberrations analysis) to invert the direction of the light rays. Thus, the reconstruction waves form an angular spectrum of plane waves (each having the diameter of the eye’s...
pupil) that emerge from the eye and are focused by the hologram onto the display’s plane; the wave at the central viewing angle is focused to the center of the display, whereas the foci of the waves with higher angles are laterally displaced.

If, as shown in Fig. 2(b), the extent of the pupil is smaller than the hologram, then a single plane wave representing a particular viewing angle illuminates only part of the overall hologram. Thus, we may define, for each viewing angle, a local hologram whose aberrations can be determined separately. These aberrations will be a function of the geometrical parameters (assuming μ = 1) of the overall hologram, and the distances x and y between the center of the local hologram and the center of the overall hologram on the axis 4 and 7, respectively. We denote $R_q$, $\beta_q$, and $\alpha_q$ as the parameters for the overall hologram, and $R_q(x,0)$, $\beta_q(x,0)$, $\alpha_q(x,0)$ for the local hologram. Since we are dealing with a very high F/No. ($d_{eye} >> R$) and high obliquity ($\sin\beta_q > 1/2$), then the dominant aberration is the astigmatism (i.e., $A_x$, $A_y$, and $A_{xy}$). In the following, we shall calculate these aberrations as a function of $x, y$.

Under the assumption of small angles, the parameters of the local holograms, along the $\xi$ axis, are

$$\sin\beta_q(x,0) \approx \sin\beta_q - \frac{x}{R_q} \cos\beta_q \approx \sin\beta_q - \frac{x}{R_q} \cos^2\beta_q, \quad (17)$$

$$\sin\beta_q(x,0) = \sin\beta_q, \quad (18)$$

$$\sin\beta_y(x,0) \approx \sin\beta_y - \frac{x}{R_y} \cos\beta_q \approx \sin\beta_y - \frac{x}{R_y} \cos^2\beta_q, \quad (19)$$

$$R_q(x,0) \approx R_q - x \sin\beta_q, \quad (20)$$

$$\frac{1}{R_q(x,0)} = \frac{1}{R_q(x,0)} = 0 \Rightarrow F(x,0) = -\frac{1}{R_q(x,0)} - \frac{1}{R_q(x,0)}. \quad (21)$$

Substituting Eqs. (17)–(19) into Eq. (2) and using the relation $\sin\beta_q = -\sin\beta_q$ and the fact that along the $\xi$ axis $\cos\alpha_q(x,0) = 1$ ($q = \alpha, r, c$), we can find that

$$\sin\beta_q(x,0) = \sin\beta_q(x,0) - \sin\beta_q(x,0) + \sin\beta_q(x,0) = \sin\beta_q(x,0) - \sin\beta_q(x,0) + \sin\beta_q(x,0) \approx \sin\beta_q - \frac{x}{R_y} \cos\beta_q + \frac{x}{R_q} \cos^2\beta_q + \sin\beta_q, \quad (22)$$

$$= -\sin\beta_q + \frac{x}{R_q} \cos\beta_q - \frac{x}{R_y}. \quad (23)$$

By exploiting Eqs. (17)–(22) we can calculate the astigmatism along the $\xi$ axis, using only the first nonvanishing order of $x$,
\[ A_{xy}(0,y) = \frac{\sin \alpha(0,y) \sin \beta(0,y)}{R_f(0,y)} - \frac{\sin \alpha(0,y) \sin \delta(0,y)}{R_q(0,y)} \]

\[ \approx \frac{\sin \beta}{R_o} [-\sin \alpha(0,y) - \sin \alpha(0,y)] \approx \frac{\sin \beta}{R_o} \frac{y}{R_{\text{eye}}} \]

\[ = \frac{\sin \beta}{R_o R_{\text{eye}}} y. \]

III. Design Procedure

Our primary goal is to reduce the dominant astigmatic aberrations \( A_x, A_y \), and \( A_{xy} \), given by Eqs. (23), (31), and (32). This can be conveniently achieved by introducing controlled compensating aberrations into the reference wavefront, i.e., using a distorted wave rather than a perfect plane wave. In this section we shall determine how the introduction of controlled aberrations influences the astigmatism as well as the other aberrations.

To generate the necessary reference wave with the controlled aberrations, we exploit an interim hologram, using a readout geometry and wavelength which differ from those of recording. The resulting aberrated wavefront then serves as the reference wave for the final hologram. The phase of the reconstructed wavefront from the interim hologram is given by

\[ \phi'_r = \phi'_x \pm (\phi'_y - \phi'_z), \]  \( \text{(33)} \)

where the superscript \( r \) denotes the parameters that are related to the interim hologram. Since \( \phi'_r \) becomes the phase of the reference wave for the final hologram (i.e., \( \phi'_r = \phi_r \)), then substituting Eq. (33) into Eq. (1), yields

\[ \phi = \phi_x - \phi_y + [\phi'_x \pm (\phi'_y - \phi'_z)]. \]  \( \text{(34)} \)

This equation implies that the final aberrations for each local hologram, \( Q'(x,y) \), \( Q = F, S, C_x, C_y, A_x, A_y, A_{xy} \), are comprised of two parts, as

\[ Q'(x,y) = Q(x,y) + Q'(x,y), \]  \( \text{(35)} \)

where \( Q(x,y) \) denotes the various aberrations of the noncorrected element [as part of it was found in Eqs. (23), (31), and (32)] and \( Q'(x,y) \) denotes the aberrations of the interim hologram.

The goal of the design is for \( Q'(x,y) \) to be as small as possible. To achieve this goal, the various aberrations of the interim hologram must compensate, as closely as possible, for the aberrations of the noncorrected element, so,

\[ Q'(x,y) = -Q(x,y). \]  \( \text{(36)} \)

It is desirable that this compensation be satisfied for each local hologram (or for each viewing angle), regardless of the distances \( x \) and \( y \).

We now consider the aberrations of the interim hologram in more detail, to determine how they can be exploited for compensating the aberrations of the final hologram. We begin by assuming that for all the waves of the interim hologram \( \alpha_q = 0 \) \( (q = o,c,r) \).

Similar to the derivation of Eqs. (17)-(21), the parameters of the local holograms along the \( \xi \) axis are

\[ \sin \delta_x'(0,x) \approx \sin \left( \delta_x - \frac{x}{R_q} \cos \delta_x \right) \approx \sin \delta_x - \frac{x}{R_q} \cos \delta_x, \]  \( \text{(37)} \)

\[ R_q'(0,x) \approx R_q - x \sin \delta_x', \]  \( \text{(38)} \)

\[ \frac{1}{R_q'(0,x)} \approx \frac{1}{R_q} + \frac{x}{(R_q')^2} \sin \delta_x'. \]  \( \text{(39)} \)

Using only the first nonvanishing order of \( x \) yields

\[ \frac{\sin \delta_x'(0,x)}{R_q'(0,x)} \approx \frac{\sin \delta_x - \frac{x}{R_q} \cos \delta_x}{R_q - x \sin \delta_x'} \]

\[ \approx \frac{\sin \delta_x}{R_q} - \frac{2x}{(R_q')^2} \sin \delta_x \cos \delta_x + \frac{x}{(R_q')^3} \sin^3 \delta_x', \]  \( \text{(40)} \)

\[ = \frac{\sin \delta_x}{R_q} - \frac{2x}{(R_q')^2} \sin \delta_x + \frac{3x}{(R_q')^3} \sin^3 \delta_x'. \]

Substituting Eqs. (39) and (40) into Eqs. (10), (13), and (15) yields

\[ A_x'(0,x) = A_x' - 2C_x x + 3D_x' x, \]  \( \text{(41)} \)

\[ F'(0,x) = F + C_x' x, \]  \( \text{(42)} \)

where \( F' \) can be readily arranged to zero, and where \( D_x' \) (which is one of the fifth-order aberrations of the interim hologram) is defined as

\[ D_x' = \sin \frac{\delta_x}{R_q} \left[ \frac{\sin^3 \delta_x}{(R_q')^2} - \sin^3 \delta_x \right]. \]  \( \text{(43)} \)

The parameters of the local holograms along the \( \eta \) axis are

\[ \cos \delta_y(0,y) = 1, \]  \( \text{(44)} \)

\[ \sin \beta_y(0,y) = \sin \beta_y', \]  \( \text{(45)} \)

\[ \sin \alpha(0,y) = -\frac{y}{R_y}, \]  \( \text{(46)} \)

\[ R_y'(0,y) \approx R_y. \]  \( \text{(47)} \)

Using only the first nonvanishing order of \( y \) yields

\[ \frac{\sin \alpha_y'(0,y)}{R_y'(0,y)} \approx \frac{y^2}{(R_y')^3}, \]  \( \text{(48)} \)

\[ \cos \delta_y(0,y) \sin \alpha(0,y) \sin \delta_y(0,y) \approx -\frac{y^2}{(R_y')^3} \sin \delta_y'. \]  \( \text{(49)} \)

Substituting Eqs. (48) and (49) into Eqs. (11), (12), (14), and (16) yields

\[ A_y'(0,y) = S_y y^2, \]  \( \text{(50)} \)

\[ A_{xy}'(0,y) = -C_x y. \]  \( \text{(51)} \)

Now, that we have derived the dominant astigmatic aberrations equations of \( A_x', A_y' \) and \( A_{xy}' \), we can compare them to the astigmatic aberrations of the final element according to Eq. (36) determine what each term of the equations must be to achieve the desired
compensation. We begin by juxtaposing Eqs. (31) and (50) to obtain
\[ S' = \frac{1}{R_s R_{eye}} \left[ \frac{1}{R_{eye}} - \frac{2}{R_o} \right] . \tag{52} \]
We then juxtapose Eqs. (32) and (51) to obtain
\[ C_x' = \frac{\sin \beta_o}{R_s R_{eye}} . \tag{53} \]
This result, namely that \( C_x' \) is not zero, unfortunately, introduces a field curvature [Eq. (42)] that must be now compensated for. This will be done by introducing a field curvature into the final element of the form
\[ F(x, \theta) = -C_x' \sin \theta . \tag{54} \]

By substituting Eq. (54) into Eq. (23) and comparing the resulting equation with Eq. (41) we find that
\[ A_x = 0, \tag{55} \]
\[ D_x = \frac{\sin \beta_o}{3R_s R_{eye}} . \tag{56} \]

There is also a constraint that the image wave of the interim hologram, which is the reference wave for the final hologram, must be a plane wave emerging at an angle conjugate to \( \beta_{eye} \), namely,
\[ \beta_i = 180^\circ + \beta_{eye}, \tag{57} \]
\[ \frac{1}{R_i} = 0. \tag{58} \]

To determine how the field curvature [Eq. (54)] can be introduced into the final hologram, we begin by noting that if the display plane is normal to the line between the center of the hologram and the center of the display, then for small viewing angles
\[ \frac{1}{R_s(x, \theta)} \approx -\frac{1}{R_s(x, \theta)}. \tag{59} \]

Equation (59) indicates that the field curvature, \( F(x, \theta) \), for this geometry is zero. A field curvature can be introduced by rotating the display plane around its vertical axis by an angle \( \gamma \), as shown in Fig. 3. We define \( \gamma \) as positive when the rotation is clockwise. After rotation, \( R_s(x, \theta) \) is modified to
\[ R_s(x, \theta) = -R_s(x, \theta) - x' \tan \gamma, \tag{60} \]
where \( x' \), the distance between the center of the image spot and the center of the display, is
\[ x' = \frac{x R_{dis}}{R_{eye} \cos \beta_{dis}} = \frac{x R_o}{R_{eye} \cos \beta_o}. \tag{61} \]
Substituting Eqs. (60) and (61) into Eq. (21) yields
\[ F(x, \theta) = -\frac{1}{R_s(x, \theta)} - \frac{1}{R_s(x, \theta)} \approx \frac{1}{R_s(x, \theta)} + \frac{1}{R_s(x, \theta)} \]
\[ \approx \frac{x' \tan \gamma}{(R_s)^2} \approx -\frac{x \tan \gamma}{R_{eye} \cos \beta_o}. \tag{62} \]

Finally, juxtaposing Eqs. (62) and (54) yields
\[ \gamma = \arctan(\sin \beta_o \cos \beta_o). \tag{63} \]

The relations that describe the relevant parameters of the interim hologram [Eqs. (52), (53), and (55)-(58)] can now be written explicitly as a set of six equations:
\[ \begin{align*}
\sin \beta'_c + \mu' \left( \frac{\sin \beta'_c - \sin \beta'_o}{R_c} \right) &= -\sin \beta'_o \\
\frac{1}{R_c} + \mu' \left( \frac{1}{R_c} - \frac{1}{R'_c} \right) &= 0 \\
\frac{1}{(R_c)^3} + \mu' \left[ \frac{1}{(R_c)^3} - \frac{1}{(R'_c)^3} \right] &= \frac{1}{R_s R_{eye}} \left( \frac{1}{R_{eye}} - \frac{1}{R_o} \right) \\
\sin \beta'_o \left( \frac{\sin \beta'_o}{R_c} \right) + \mu' \left[ \sin \beta'_o \left( \frac{\sin \beta'_o}{R_c} \right) - \sin \beta'_o \left( \frac{\sin \beta'_o}{R'_c} \right) \right] &= \sin \beta'_o R_s R_{eye} \\
\sin \beta'_o \left( \frac{\sin \beta'_o}{R_c^2} \right) + \mu' \left( \frac{\sin \beta'_o}{R_c^2} - \frac{\sin \beta'_o}{R'_c^2} \right) &= \frac{\sin \beta'_o}{3R_s R_{eye}}.
\end{align*} \tag{64} \]

We see that for these six equations there are seven variables—\( R'_c, \beta'_o \), \( \beta'_c \), \( \mu' \), \( q = o, r, c \) and \( \mu' \). To solve for these variables, we find it convenient to let \( \mu' \) be a free parameter. We set \( \mu' \) to be that value which minimizes the higher orders of \( x \) and \( y \) for the aberrations \( A_x, A_y, X, A_{xy}, \) and \( F \). It should be noted that here we have been limited to seven variables because we used only one interim hologram in the recording process. In general, it is possible to increase the number of variables by introducing recursively additional interim holograms in the reference waves, as well as in the object wave.

**IV. Design Illustration**

The recursive design technique will now be applied for designing and recording a combiner and imaging lens for a helmet display (Fig. 2) having the following parameters: \( R_{dis} = 100 \text{ mm}, \beta_{dis} = 40^\circ, R_{eye} = 70 \text{ mm}, \beta_{eye} = 0^\circ \). A circular aperture of \( d_{eye} = 4 \text{ mm} \) diam. was chosen for the eye’s pupil. Accordingly, the recording parameters of the noncorrected element are: \( R_o = 100 \text{ mm}, \beta_o = 40^\circ, R_r = \infty \) and \( \beta_r = 180^\circ \). By substituting these values into Eq. (64) we solved for the parameters of the interim hologram, to obtain
\[ R'_c = \frac{-88.6}{R_c} \text{ mm}, \beta'_c = 12.8^\circ, \]
\[ R'_c = \frac{194.7}{R_c} \text{ mm}, \beta'_c = 71.2^\circ. \]

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We then recorded an interim hologram from which the aberrated reference wave for the final holographic element was derived. The rotation angle of the display plane, according to Eq. (63), is

$$\gamma = 26^\circ.$$  

(66)

We calculated the aberrations as a function of a horizontal and vertical viewing angle, ranging to an overall field of vision (FOV) of $\pm 8^\circ$ for a corrected element as well as a noncorrected element (recorded with unaberrated reference wave). These aberrations were converted to units of mrad by exploiting the relation

$$\frac{\lambda x}{2\pi} \frac{\partial}{\partial x}$$

for each aberration.\textsuperscript{11} The calculated result of the various aberrations are shown in Figs. 4 and 5; note that the aberration scale of $Ax$ and $Axy$ for the noncorrected element [Figs. 4(e) and (f)] is greater by a factor of 10 than the scale for the other aberrations. As shown, the dominant aberration of the noncorrected element is the $Ax$ [Figs. 4(e)] and $Axy$ [Fig. 4(f)], whereas the contribution from the other aberrations is negligible. For the corrected element the astigmatism is significantly improved. Although $Cx$ is now the dominant aberration [Fig. 5(c)], it is still much smaller than the astigmatism of the noncorrected element. Thus, the total amount of aberrations for the corrected element is significantly smaller than for the noncorrected element.

We also calculated the size and shape in the display plane that the eye sees through the hologram for each viewing angle. This was done by a ray tracing analysis, where the rays were traced from the pupil through the hologram onto the display plane. Figures 6 and 7 show the calculated spot diagrams covering a FOV up to $\pm 8^\circ$. Figure 6 shows the spot diagram from the noncorrected element. In each part of the figure, spots derived from four different viewing angles, with an angular separation of 8 mrad, are drawn; $\theta_x$ and $\theta_y$. 

$$R = 79 \text{ mm}, \; \beta = 34^\circ,$$

(65)

$\mu = 0.77$. 

$\gamma = 26^\circ$. 

(66)
the field curvature

Fig. 4. The aberrations of the noncorrected element. Aberrations given in milliradians; \( h_{\text{angle}} \) denotes horizontal angle and \( v_{\text{angle}} \) denotes vertical angle, both \( h_{\text{angle}} \) and \( v_{\text{angle}} \) are in degrees. See facing page.

denote the area in the FOV in the horizontal and vertical direction, respectively. It can be deduced from these results that the resolution of the imaging element in the horizontal axis is worse than 8 mrad at the edges of the FOV. Figure 6 shows the spot diagram for the corrected lens; note the factor of 6 between the scales of Fig. 6 and Fig. 7. For the corrected lens, the angular separation between the viewing angles was 0.8 mrad. As can be deduced from these results, the resolution of the corrected lens is now better than 0.8 milliradians over the FOV.

To verify our design and calculations, we recorded the necessary interim hologram and then the final holographic element. The reconstructed aberrated wavefront from the plane of the interim hologram was transferred by means of a telescope to the plane of the final holographic element, according to the arrangement shown in Fig. 8. For comparison, we also recorded a conventionally designed noncorrected element. These elements were then tested experimentally at six different areas of the FOV by introducing plane waves from a rotating mirror at the location of the pupil and checking the spots at the display plane. The spots at the display plane were photographed and the results are shown in Figs. 9 and 10; note the factor of 2 between the scales in the Figures. Figure 9 shows the experimental results for the noncorrected element, where only one plane wave was used for each area. As expected the central spot, at \( \theta_x = \theta_y = 0 \), is very small and we measured that it is a diffraction limited spot. However, the spot sizes increase significantly at the edges of the FOV. Figure 10 shows the experimental results for the corrected element. Here, four adjacent plane waves with angular separation of 0.8 mrad were used for each area of the FOV. As shown all four spots can be resolved uniformly over the entire range of FOV, indicating that the resolution is at least 0.8 mrad. The improvement in performance of the corrected element is evident.

V. Constructing an Element with High Diffraction Efficiency

So far we dealt with the aberrations and image geometry that depend on the 2-D grating function of the holographic element. As a result, the diffraction efficiency is low at the edges of the FOV where the Bragg relation is not satisfied. We shall now design the element so that, in addition to having low aberration, the diffraction efficiency, which depends on the 3-D volume distribution of the grating, is high over the
entire FOV. If we were solely interested in obtaining a high diffraction efficiency, we could have recorded the element with two spherical waves, one of which converges to (or diverges from) the center of the pupil, as shown in Fig. 11. In such a case the relevant geometry and parameters for our design would be $R_B = -70$ mm, and $\beta_B = 180^\circ$. Then, in order to ensure that the imaging parameters will be the same as for the element that was designed earlier, $R_B = -233$ mm and $\beta_B = 40^\circ$. The parameters having a superscript B belong to the optimal recording waves that fulfill the Bragg relation. It is evident, for the geometries shown in Figs. 2(b) and 11 that the central ray at each viewing angle satisfies the Bragg relation. Consequently, since $d_{\text{eye}} \ll R_{\text{eye}}$, the rays around each central ray will also satisfy the Bragg relation. Unfortunately, although such an element has high diffraction efficiency, it would contain relatively large aberrations even at the center of the FOV.

To obtain an element having low aberrations as well as high diffraction efficiency, it is possible to incorporate the optimal holographic low aberration element into a final element with the same grating function and with high diffraction efficiency. We start with the optimal grating function of the corrected element,

$$\psi_H = \psi_0 - \phi_c - \phi_0^B,$$  \hspace{1cm} (67)

where $\phi_0^c, \phi_0$, and $\phi_c$ are the same as before. The incorporation of optimization and high diffraction efficiency into the final element can be achieved with the aid of an intermediate hologram as shown in Fig. 12. The intermediate hologram is recorded with a planar reference wave, having the phase $\phi_0^{\text{int}}$ and an object wave that is derived from the diffracted negative first-order of the optimized hologram $H$. If the optimized hologram is reconstructed with a wavefront having a phase $\phi_c = -\phi_0^B$, then the reconstructed wavefront, which serves as the object wavefront for the intermediate hologram, is

$$\phi_i = -\phi_0^B - \phi_H^{\text{int}}.$$

The configuration for recording the final element $H'$ is shown in Fig. 13 (the prime denotes all the parameters which refer to the final hologram). The intermediate hologram is reconstructed with a conjugate plane wave (i.e., $\phi_c^{\text{int}} = -\phi_r^{\text{int}}$), so the phase of the
Fig. 5. The aberrations of the corrected element. Aberrations given in milliradians; h.angle denotes horizontal angle and v.angle denotes vertical angle, both h.angle and v.angle are in degrees. See facing page.

Fig. 6. The spot diagram for the noncorrected element.

Fig. 7. The spot diagram for the corrected element.

reconstructed wavefront is precisely $-\phi_i$ at the plane of $\mathcal{H}'$ which is the original location of $\mathcal{H}$; thus $\phi'_o = -\phi_i$. The reference wave for $\mathcal{H}'$ is the conjugate of the wave that was used for reconstructing $\mathcal{H}'$, thus

$$\phi'_o = \phi^B.$$  (69)

Fig. 8. Geometry for recording a corrected holographic element.
Substituting Eqs. (68) and (69) into Eq. (67), yields the grating function for \( \mathcal{H} \) as

\[
\phi_{\mathcal{H}} = \phi^o - \phi^o - \phi^i - \phi^i = \phi^o + \phi_i^o - \phi_i^o.
\]  

(70)

This grating function was, of course, recorded with reference and object wavefronts that are appropriate for an efficient hologram. Specifically, Eq. (69) implies that the parameters for the reference wave are

\[
R_s = R^o,
\]  

(71)

\[
\beta_s = \beta^o.
\]  

(72)

Similarly, from Eq. (68), and using the fact that
and that \( \sin \beta_0 - \sin \beta_r = \sin \beta_0^B - \sin \beta_r^B \), we find that

\[
\frac{1}{R_o} - \frac{1}{R_r} = \frac{1}{R_o^B} - \frac{1}{R_r^B}.
\]

Clearly then, the conditions of optimal grating function and high diffraction efficiency were incorporated into the final hologram \( \mathcal{H}' \).

Figures 14 and 15 shows the calculated diffraction efficiencies of the holograms \( \mathcal{H} \) and \( \mathcal{H}' \) as a function of the horizontal and vertical viewing angle, over a FOV of \( \pm 8^\circ \). For the calculation we assumed that the hologram thickness is 15 \( \mu \text{m} \), the average refractive index is 1.5, and the refractive index modulation is 0.04. As shown, the diffraction efficiency for \( \mathcal{H} \) decreases rapidly after 5\(^\circ\), whereas for \( \mathcal{H}' \) it remains high over the entire FOV of \( \pm 8^\circ \).

Figure 16 shows the experimental diffraction efficiencies of the holograms \( \mathcal{H} \) and \( \mathcal{H}' \) as a function of the horizontal viewing angle (the vertical viewing angle is...
0). The hologram was recorded in 15-μm thick dichromated gelatin which was prepared from Kodak 649/F plates, and the exposure times were calculated to achieve 0.04 refractive index modulation. As shown, the diffraction efficiency for \( H \) decreased to 0 at the edges of the FOV, whereas for \( H' \) it remains >80% over the entire FOV.

VI. Concluding Remarks

We have designed and recorded a corrected element for holographic helmet display, having low aberrations and high diffraction efficiency over a relatively wide FOV. Although we used only one interim hologram, we obtained resolution of 0.8 mrad and a diffraction efficiency of 80% over a FOV of ±8°, we believe that by adding more interim holograms to the recording waves, it is possible to improve the performance of the final element, so as to obtain even wider FOV and better resolution.

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References


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referred to above but also notes on units and conversion factors, a calculation of electromagnetic wave field energy, and a useful set of thirty-three worked problems. There is a short bibliography of other introductory books and related review papers (some as late as 1987) and a comprehensive index. The figures are plentiful and clear.

One can always find points about which to quibble: for example, I was disconcerted to see symbols such as \( r \) and \( \Gamma \) for the refractive and absorptive indices, even though I recognize that the more conventional use of \( n \) and \( k \) might confuse students becoming acquainted with their use for quantum numbers, carrier concentrations, wave vectors, and so on. On the other hand, it is a pleasant change to read a text where the refractive index is clearly defined with reference to the phase velocity of light. This helps to alleviate the stress experienced by those who subsequently discover that the index can have values well below unity.

It must be noted that, as the title indicates, there is no attempt to make this book cover all aspects of solid state physics even at an introductory level. Topics such as the thermal properties or even crystal structure require a supplementary text. However, the average student should find this book an attractive and refreshingly readable introductory text on electrons in solids. At $39.50, this hard cover edition is a good value.

DAVID M. ROESSLER


This book will be referred to often. The editors and authors have done an excellent job in presenting the state of the art in the field of imaging techniques in biology and medicine. It is a showpiece, presenting the many aspects of a complex application in a concise, lucid, and easily readable way. Of importance is the perspective we are provided by users of imaging systems in achieving their objectives and how well these objectives are achieved with present day technology and techniques. The reader will appreciate the problems that need addressing as well as the synergism that must exist between the imaging designers and the biologist or medical personnel. The book serves as an introduction to representative uses and principles of imaging methodologies in these disciplines. Examples are given of the resulting images for the various modalities, e.g., ultrasound, autoradiography, magnetic resonance, nuclear medicine. In this respect, the quality of the photoreproduction is good. The technical difficulties, such as spatial resolution and noise, are related to the visual interpretation, feature extraction, and quantitation measure of the physiological processes.

Chapter 1, by Loats, Pittenger, Tucher, and Unnerstall, sets the tone. A generic image processing system is discussed as well as a review of several applications and processing algorithms. Chapter 2, by Jonathan Links, addresses Digital Image Processing in Nuclear Medicine. Percival McCormack is the author of Chap. 3, Fundamental Principles of Imaging by Ultrasound. Chapter 4, Radiotransfer Methodology, by Robert Eng, explains the use of radioisotopes (radionuclides) that are integral to many of the techniques described in this book, e.g., autoradiography and tomography. Although the orientation is toward chemistry and biology, I believe that the nonspecialist can readily follow the exposition. Principles of Quantitative Autoradiograph in Biomedical Research, Chap. 5, discusses the quantitative analysis of receptor autoradiograms. James Unnerstall explains the role of radiotracers (Chap. 4) in this modality. The clarity of writing plus the reproductions are an aid to understanding for the nonspecialist. Chapter 6, by Thomas Hinz,