Recitation 5
Root locus - Cont.

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This recitation is partially based on problem 10.14 in Razavi. The circuit is given on Fig. 1. The small signal equivalent circuit is shown on Fig. 2. See

![Circuit Schematics](image)

Figure 1: Circuit schematics.

<table>
<thead>
<tr>
<th>$r_{01}$</th>
<th>10kΩ</th>
<th>$g_{m1}$</th>
<th>5.2 mA/V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{02}$</td>
<td>5kΩ</td>
<td>$g_{m2}$</td>
<td>2.8 mA/V</td>
</tr>
<tr>
<td>$r_{03}$</td>
<td>1MΩ</td>
<td>$g_{m3}$</td>
<td>0.16 mA/V</td>
</tr>
<tr>
<td>$C_1$</td>
<td>77fF</td>
<td>$C_2$</td>
<td>80fF</td>
</tr>
<tr>
<td>$C_3$</td>
<td>106fF</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Values of the elements of the circuit on Fig. 2.
1.1 Topological analysis

The feedback type is PIPO, and therefore the transimpedance gain is stabilized. The Kirchhoff matrix is

\[
\begin{pmatrix}
  i_1 \\
i_o
\end{pmatrix} = \begin{pmatrix}
I_{11} & B \\
A & I_{22}
\end{pmatrix} \begin{pmatrix}
V_1 \\
V_o
\end{pmatrix},
\]

where

\[
I_{11} = \frac{i_1}{V_1|_{V_o=0}} = sC_1 + g_{03},
\]

\[
B = \frac{i_1}{V_o|_{V_1=0}} = g_{m3},
\]

\[
A = \frac{i_o}{V_1|_{V_o=0}} = -\frac{g_{m1}g_{m2}r_{01}}{1 + sr_{01}C_2},
\]

\[
I_{22} = \frac{i_o}{V_2|_{V_1=0}} = g_{02} + sC_3.
\]

\[LT = -K \frac{1}{(s + \omega_1)(s + \omega_2)(s + \omega_3)},\]

where

\[
K = \frac{g_{m1}g_{m2}g_{m3}}{C_1C_2C_3},
\]

and \(\omega_1 = 1/(r_{01}C_2) \approx 1.25 \times 10^9 \text{ sec}^{-1}\), \(\omega_2 = 1/(r_{02}C_3) \approx 1.89 \times 10^9 \text{ sec}^{-1}\) and \(\omega_3 = 1/(r_{03}C_1) \approx 1.3 \times 10^7 \text{ sec}^{-1}\).

1.2 Root Locus

The root locus is shown on Fig. 3. The gain is \(K = 3.6 \times 10^{10} \text{ sec}^{-3}\) and the asymptotes origon is \(b_0 = 1.05 \times 10^9 \text{ sec}^{-1}\). The closed loop system is unstable, see Fig. 4.
1.3 Compensation capacitor

In order to make the closed loop system stable one may reduce the gain of $LT$, so it does not encircle -1. This can be achieved by introducing a "compensation capacitor" $C_c$ between the gate and the drain of $M_1$. This will create a dominant low frequency pole, reduce the gain and make the frequency response of the amplifier less dependent on parasitic capacitances $C_1 - C_3$. We will estimate the impact of introducing large $C_c$ (several picofarads) by using its Miller equivalent:

$$C_1 \rightarrow C_1 + C_c(1 + g_{m1}r_{01}),$$

$$C_2 \rightarrow C_2 + C_c \frac{g_{m1}r_{01} + 1}{g_{m1}r_{01}}.$$

For example, if we use $C_c = 3\ \text{pF}$ the new poles are $\omega_1 \approx 0.33\ \text{sec}^{-1}$ and $\omega_3 \approx 6.29 \times 10^3\ \text{sec}^{-1}$, $\omega_2$ remains unchanged. Also, $K = 4.5 \times 10^{17}\ \text{sec}^{-3}$ and the asymptotes' origin $b_0 = 6.3 \times 10^8\ \text{sec}^{-1}$. The new root locus can be seen on Figs. 5,6. The closed loop system is now stable, see Fig. 7. The Bode magnitude plot of both closed loop systems is shown on Fig. 8. Note that the uncompensated system is unstable, therefore its Bode plot cannot be used to predict its steady state frequency response, which diverges.
Figure 4: Matlab generated impulse response and pole-zero map for the uncompensated closed loop system. The impulse response diverges and there is a pair of RHP poles. Also, Nyquist graph of $-LT$ can be seen to encircle -1. The closed loop system is unstable.

Figure 5: The root locus of the compensated circuit.
Figure 6: The zoom in of root locus of the compensated circuit at the breakout point.

Figure 7: Matlab generated impulse response and pole-zero map for the compensated closed loop system. The impulse response is stable and there are no RHP poles. Also, Nyquist graph of $-LT$ does not encircle -1. The closed loop system is stable.
Figure 8: Matlab generated Bode plots of both uncompensated and compensated closed loop systems.