Subject 3

Stability
Frequency response of amplifier

Common term to all parameter set terms is: \((1-\text{LT})\)

for example: \(T_f = \frac{1}{B} \cdot \frac{-\text{LT}}{1-\text{LT}}\)

It is important to explore \((1-\text{LT})\)
The zeros of \(1-LT(s)=0\) are the poles of all transfer functions that belong to the parameter set.

The frequency response analysis of amplifiers is essential for the design of stable amplifiers.

Controllable output and input impedances, monotonic responses control of noise performance ….
We shall cover 3 important techniques for the analysis of frequency response:

1. Bode diagram
2. Nyquist diagram
3. Root Locus
Example 1

Consider \[ LT = -\frac{a_0 \cdot B}{1 - \frac{S}{P_1}} \]

Where \( P_1 < 0 \) and both \( a_0, B \) are not functions of \( S \)

\[ T_f = \frac{1}{B} \cdot \frac{-LT}{1 - LT} = \frac{a_0}{1 + a_0 \cdot B} \cdot \frac{1}{1 - \frac{S}{P_1 (1 + a_0 \cdot B)}} \]

where \( LT(0) = -a_0 \cdot B \)

\[ T_f = \frac{a_0}{1 - LT(0)} \cdot \frac{1}{1 - \frac{S}{P_1 (1 + a_0 \cdot B)}} \]
Observation

- LT includes a single pole at the RHP
- Closed loop Transfer function includes a single pole at the LHP \( P_c = P_1 \cdot [1 - LT(0)] \)
  - It increases proportional to \([1 - LT(0)]\)
- The low frequency gain is \( A = \frac{a_0}{1 - LT(0)} \)
- The low frequency gain decreases proportional to \([1 - LT(0)]\)
The following relation exist:

\[ P_C \cdot A = a_0 \cdot p_1 \]

This product is the Gain bandwidth product.
Gain bandwidth product theorem

Given:
- An amplifier with gain $A(s)$ that includes poles at the LHP only.
- A feedback network $B$ that is not a function of $(s)$
- $A(s)$ is given by:

$$A(s) = \frac{A(0)}{[1 + s \cdot A(0) \cdot \tau_0] \cdot \prod_{i=1}^{n-1} (1 + s \cdot \tau_i)}$$
\[ LT(s) = \frac{A(0) \cdot B}{\left[ 1 + s \cdot A(0) \cdot \tau_0 \right] \prod_{i=1}^{n-1} (1 + s \cdot \tau_i)} \]

- \( A(0) \cdot B >> 1 \)
- \( \tau_0 >> B \cdot \sum_{i=1}^{n-1} \tau_i \)

**Result**

- \( \frac{1}{\tau_0} \) is the gain bandwidth product of the amplifier
- \( \frac{1}{A(0) \cdot \tau_0} \) is the frequency of the dominant pole
- \( \frac{B}{\tau_0} \) is the bandwidth in closed loop

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For frequency range of \( s \gg \frac{1}{A(0) \cdot \tau_0} \), the LT can be approximated to

\[
LT(s) \approx - \frac{B}{s \cdot \tau_0 \cdot \prod_{i=1}^{n-1} \left(1 + s \cdot \tau_i \right)}
\]
Bode diagram

Bode diagram is a graph ofenly plotted with logarithmic scale that plots a complex transfer function vs. frequency. It consists of two graphs one for phase and one for magnitude.
Bode plot of example 1

\[ 20 \log a_0 \]

\[ 20 \log \left[ \frac{a_0}{1-LT(0)} \right] \]

\[ 20 \log \left[ 1 - LT(j\omega) \right] \]

\[ P_1 \]

\[ P_1 \left[ 1 - LT(0) \right] \]
Nyquist diagram is a two dimensional graph that plots imaginary value of a transfer function vs. real value of the same transfer function where changes from 0 to $\omega \rightarrow \infty$. 
Nyquist plot of example 1

\[ L = |1 - LT(j\omega)| \]
Stability observed from Nyquist plot

If \( |LT(j\omega)| > 1 \) when \( \angle \{ -LT(j\omega) \} = \pi \)

the amplifier is not stable.

EXAMPLE

\[
LT(s) = -\frac{a_0 \cdot B}{\left(1 - \frac{s}{P_1}\right) \cdot \left(1 - \frac{s}{P_2}\right) \cdot \left(1 - \frac{s}{P_3}\right)}
\]
\[
\text{Im}\{-LT(j\omega)\}
\]

\[
\text{Re}\{-LT(j\omega)\}
\]

\((-1,0)\)

Stable

Unstable
Bode description

\[ 20 \log |LT| \]

\[ \omega \]

\[ p_1 \quad p_2 \quad p_3 \]

\[ \log \omega \]

\[ \frac{-\pi}{2} \]

\[ -\pi \]

\[ \frac{-3\pi}{2} \]
Phase margin and gain margin

- **Gain margin** is the difference between the value of \( |LT(s)| \) and 1 when \( \angle LT(s) = -\pi \).

- **Phase margin** is the phase difference between \( \angle LT(j\omega) \) and \(-\pi\) when \( |LT(j\omega)| = 1 \).